PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS

Volume 18, Part 1, *NONLINEAR FUNCTIONAL ANALYSIS*

Part 2, *NONLINEAR OPERATORS AND NONLINEAR EQUATIONS OF EVOLUTION IN BANACH SPACES*

A symposium on Nonlinear Functional Analysis held in Chicago in April 1968 has resulted in the publication of these two volumes edited by Felix E. Browder. The first part, published in 1970, contains papers contributed by the following speakers:

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The second part, just published in 1976, contains the book-length text of a paper by Felix E. Browder composed in its entirety during 1968. It is a detailed treatment of most of the major branches of nonlinear functional analysis as they had developed up to that time. The manuscript of this work has had wide circulation in mimeographed form and has been referred to in a considerable number of research papers. It is now at last available.

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OF THE AMERICAN MATHEMATICAL SOCIETY
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MAY 1974

"Very important mathematics is being spoken here"

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MATHEMATICAL DEVELOPMENTS ARISING FROM HILBERT PROBLEMS

EDITED BY
FELIX E. BROWDER

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